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THE MECHANISM OF NUCLEATE BOILING HEAT TRANSFER

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Abstract—In nucleate boiling a bubble created at nucleation site on a heating surface grows, leaves the surface and rises. The fluid motion induced in the thermal boundary layer during this process is calculated and the heat flux carried by this liquid motion to a nucleation site is obtained. The heat flux thus calculated is equal to that transferred from the heating surface to the liquid by conduction and to the latent heat carried away by the bubble per unit time. From these relations the following theoretical formula is obtained.

$$\Delta\theta = 0{\cdot}114\, n^{-\frac{1}{4}}\, q^{\frac{2}{3}}$$

where $\Delta\theta$ is a temperature difference between the temperature of heating surface and the saturation temperature, n is the number of nucleation sites per unit area and q is average heat flux. This closely resembles in form the following experimental formula which was obtained through measurement by Nishikawa:

$$\Delta\theta_{\text{exp}} = 0{\cdot}448\, n^{-\frac{1}{6}}\, q^{\frac{2}{3}}.$$

The numerical values computed from these two formulas are in fairly good agreement with each other.

NOMENCLATURE

- x, radial horizontal distance from a nucleation site, m;
- x_ν, radial horizontal distance from the nucleation site ν, m;
- y, vertical distance from a heating surface, m;
- r, radius of a growing bubble, m;
- r_0, radius of a rising bubble, m;
- δ, thickness of thermal boundary layer, m;
- δ_*, vertical displacement of a fluid particle which exists at first on the outer edge of a thermal boundary layer, during a period T, m;
- e, a distance shown in Fig. 8, m;
- h, a distance shown in Fig. 8, m;
- ρ_1, distance from a source (or doublet) to an arbitrary point in the liquid, m;
- ρ_2, distance from the reflected image of a source (or that of a doublet) to an arbitrary point in the liquid, m;
- l, a distance shown in Fig. 10, m;
- L, distance between two neighbouring bubbles, m;
- S, area of a heating surface, m^2;
- t, time, h;
- T, time interval from an instant of the creation of a bubble nucleus to that of the succeeding bubble nucleus at the same nucleation site, h;
- $f = \frac{1}{T}$, frequency of bubble formation, 1/h;
- n, number of nucleation sites per unit area of a heating surface, 1/m^2;
- $N = nS$, number of nucleation sites on a heating surface;
- U_0 rising velocity of a bubble, m/h;
- u, x-component of liquid velocity, m/h;
- v, y-component of liquid velocity, m/h;
- ϕ, velocity potential, m^2/h;
- ψ, stream function, m^2/h;
- $\Delta\theta$, difference between the temperature of heating surface and the saturation temperature of liquid, °C;
- q, heat flux from heating surface, kcal/m^{2}h;
- λ, thermal conductivity of liquid, kcal/mh degC;

γ, specific weight of liquid, kg/m^3;
c_p, specific heat of liquid, kcal/kg degC;
γ_v, specific weight of vapour, kg/m^3;
k, latent heat of liquid, kcal/kg.

1. INTRODUCTION

THERE are many experimental data [1–3] concerning nucleate boiling heat transfer and also many theoretical works [3–7] executed to clarify this problem. It may be said, however, that the phenomena remain far from being understood. The nucleate boiling is a kind of problem with non-reproducible character. The non-reproducible character consists in the creation of bubble-nuclei. As the bubble-nuclei are created in very fine irregular cavities on heating-surfaces, their creation cannot be treated from the macroscopic point of view and there is no alternative but to treat it as phenomena governed by chances. If we put the creation of bubble-nuclei out of consideration, the phenomena taking place thereafter—growing and rising of bubbles—will not have the non-reproducible character any longer and can be treated by the physics of continuum. Even the phenomena which occur after the appearance of bubble-nuclei have not yet been thoroughly understood. The author analysed these phenomena theoretically. The author proposes a hydrodynamic model as shown in the following. During the process in which a bubble once created grows, leaves the heating surface and rises up, the liquid surrounding the bubble moves. In the vicinity of the heating surface, i.e. in the thermal boundary layer, the liquid flows gradually in the direction of nucleation site with the motion of bubbles being created one after another at the nucleation site. The heat carried by this liquid flow arrives at the nucleation site, evaporates the liquid and makes a bubble at the site grow. Using this model, the mechanism of nucleate boiling heat-transfer will be revealed theoretically. The obtained results are in fairly good agreement with the experimental data measured by Nishikawa [3].

2. THEORY OF NUCLEATE BOILING HEAT TRANSMISSION

In this chapter the mechanism of nucleate boiling heat transfer is considered. For the consideration of heat transmission itself, sufficient knowledge about the motion of liquid induced by the growing and rising of bubbles is required. The theory concerning this induced motion of the liquid is, however, given in the appendix for the sake of avoiding any complication. Referring to the results in the appendix, only the theory concerning the heat transmission proper will be described in the text.

2.1 *Fundamental relations*

It is a well-known fact that a heating surface is covered with a thermal boundary layer [1, 8, 9]. Heat is supplied from the heating surface to the thermal boundary layer through thermal conduction. At nucleation sites on the heating surface bubbles are created, grow, leave the heating surface and rise. As will be mentioned in the appendix, a flow is induced in the liquid by the motion of bubbles. The liquid in the thermal boundary layer also, of course, flows and the heat contained in the thermal boundary layer is carried with it to the nucleation sites. The heat transferred from the heating surface to the liquid through thermal conduction balances with that carried by the liquid flow in the thermal boundary layer. Consequently the thickness of thermal boundary layer is kept constant approximately. The heat carried to a nucleation site evaporates the liquid and grows bubbles. Based on this model the fundamental equations are worked out.

(i) Heat given from the heating surface to the thermal boundary layer.

As shown in Fig. 1 we denote the mean thickness of the thermal boundary layer by δ, the

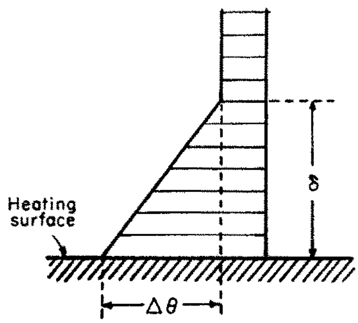

FIG. 1. Temperature distribution in a thermal boundary layer.

difference between the temperature of the heating surface and the saturation temperature by $\Delta\theta$ and the thermal conductivity of the liquid by λ. Assuming the linear change of temperature in the thermal boundary layer for the sake of simplicity, the heat flux supplied from the heating surface can be expressed as follows:

$$q = \lambda \frac{\Delta\theta}{\delta}. \tag{1}$$

δ varies in the course of time but the amount of its variation is known to be negligibly small through the calculation in the appendix (c.f. Appendix, A.5).

(ii) Heat flow in the thermal boundary layer.

The liquid motion in the process of a bubble growing and rising is considered. This motion is somewhat complicated. The locus of a liquid particle which existed at first on the outer edge of a thermal boundary layer (P in Fig. 2) is treated. In the first period in which the bubble grows, the liquid particle moves from P to Q. In the next period in which the bubble rises, the

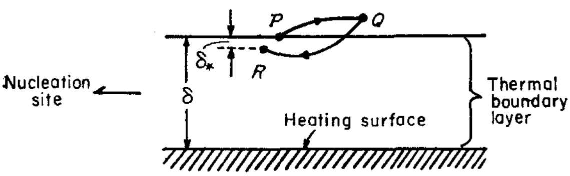

FIG. 2. The locus of a liquid particle which existed at first on the outer edge of a thermal boundary layer.

particle moves back from Q to R. R is nearer just a little to the nucleation site than P. The vertical height of R is slightly lower than that of P, if we disregard the extreme vicinity of the nucleation site. The vertical downward displacement in one cycle† $T = 1/f$ is denoted by δ_*. We have considered heretofore about an isolated nucleation site. There are, however, many nucleation sites on usual heating surfaces and the contribution of these nucleation sites to the liquid motion in the thermal boundary layer is to be superposed. In A.5 in the appendix the superposed vertical displacement is calculated. In the calculation of this superposed displacement we assumed that nucleation sites are distributed on the heating surface in the equi-lateral triangular pattern as shown in Fig. 10. The size of bubbles r_o and the bubble-forming frequency f in all the nucleation sites are assumed to be equal. The phases of bubble-formation at each nucleation sites are allowed to be different. The mean value of δ_*/δ over the heating surface is taken and is shown as follows:

$$\frac{\delta_*}{\delta} = c_1(n\, r_o^2)^m, \quad c_1 = 5{\cdot}5, \quad m = 1{\cdot}16 \tag{2}$$

where n is the number of nucleation sites per unit area and r_o is the radius of bubbles rising in the liquid.

We now compare the state in the thermal boundary layer at a given instant with that after one cycle T. Since the phenomena are periodic, the two states must be same. Heat being supplied from the heating surface, the outer edge of thermal boundary layer itself returns to the original level P (in Fig. 2) after one cycle T, though the liquid particle which was at first at P is displaced downward and does not come back to the original level. From Fig. 3 the heat

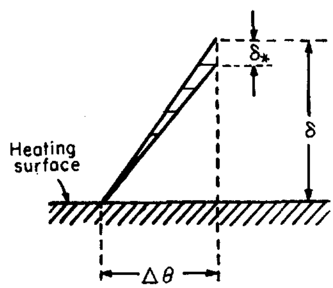

FIG. 3.

supplied from the heating surface is expressed as follows:

$$q = \frac{1}{2}\, c_p\, \gamma \delta_*\, \Delta\theta\, f \tag{3}$$

where c_p and γ are specific heat and specific weight of the liquid respectively and f is the frequency of bubble-formation.

(iii) Evaporation.

As mentioned above, the liquid in the thermal boundary layer flows to the nucleation sites. Heat in company with the liquid motion also

† If the interaction between succeeding bubbles is ignored, δ_* is equal to the vertical displacement in the case of a single bubble moving away from the heating surface.

moves and at last arrives at the nucleation sites, where the heat vaporizes the liquid and makes bubbles grow.

The latent heat consumed in the bubble-formation per unit time and per unit area of the heating surface is expressed as follows:

$$q = \frac{4\pi}{3} r_o^3 \gamma_v k n f \tag{4}$$

where γ_v and k are the specific weight and the latent heat of the vapour. In the actual boiling, bubbles continue to grow after leaving the heating surface. In our model, however, we assumed for simplicity that any growth of bubbles does not occur in the rising process.

(iv) A relation found by Jakob.

Jakob [1] found previously the following relation.

$$f r_o = c_2, \qquad c_2 = 200 \text{ m/h}. \tag{5}$$

Qualitative explanation of this relation seems to be possible through the theoretical consideration of the liquid flow due to bubble-motions. The numerical value of c_2 determinated by Nishikawa [10] is used.

2.2 *The theoretical result and its comparison with experimental data*

Five equations (1–5) are obtained in the last subsection. Independent variables involved in these equations are seven. Eliminating four variables from the five equations, a relation containing three variables can be obtained. If one more equation could be obtained, the relation between the two variables, e.g. q and $\Delta\theta$, would be obtained. The very equation which is not yet obtained would indicate the microscopic condition of the creation of bubble-nuclei at the fine irregular cavities on the heating surfaces. As mentioned in the introduction, the phenomena of nuclei-creation have a non-reproducible character and cannot be treated by the physics of continuum. Without touching this condition, we proceed with the analysis. Eliminating r_o, δ, δ_* and f, the following formula is obtained

$$\Delta\theta = \left(\frac{c_1 c_2 c_p \gamma \lambda}{2}\right)^{-\frac{1}{2}} \times \left(\frac{4\pi c_2 \gamma_v k}{3}\right)^{(2m-1)/4} n^{-\frac{1}{4}} q^{(5-2m)/4}. \tag{6}$$

From (2) and (5) $m = (7/6)$, $c_1 = 5{\cdot}5$, $c_2 = 200$ m/h. Substituting them into (6) we have

$$\Delta\theta = 0{\cdot}4\,(c_p \gamma \lambda)^{-\frac{1}{2}} (\gamma_v k)^{\frac{1}{3}} n^{-\frac{1}{4}} q^{\frac{2}{3}}. \tag{7}$$

Considering the nucleate boiling of water under atmospheric pressure, we put $\lambda = 0{\cdot}558$ kcal/mh degC, $c_p = 1$ kcal/kg degC, $\gamma = 1000$ kg/m^3, $\gamma_v = 0{\cdot}6$ kg/m^3 and $k = 540$ kcal/kg, into (7) and obtain

$$\Delta\theta = 0{\cdot}114\, n^{-\frac{1}{4}} q^{\frac{2}{3}}. \tag{8}$$

For the purpose of comparison with this theoretical result, the following experimental formula is made by the author from Nishikawa's measurements [3, p. 233, Fig. 13].

$$\Delta\theta_{\text{exp}} = 0{\cdot}0448\, n^{-\frac{1}{6}} q^{\frac{2}{3}}. \tag{9}$$

The definition of n in Nishikawa's paper is not the same as that in the present paper. n used in the equation (9) is one conforming to our definition.

The two formulas (8) and (9) are closely similar in form. Though the indices of n are slightly different, those of q coincide perfectly with each other. Their ratio is given by the following formula

$$\frac{\Delta\theta}{\Delta\theta_{\text{exp}}} = 2{\cdot}55\, n^{-1/12}. \tag{10}$$

Equation (10) is shown in Fig. 4; it has the values not far from 1 over a very wide range of n. The experimental range of n by Nishikawa is about 10^2 to 10^5.

Thus the mechanism of the nucleate boiling heat transfer can be explained well by the hydrodynamic model established by the author.

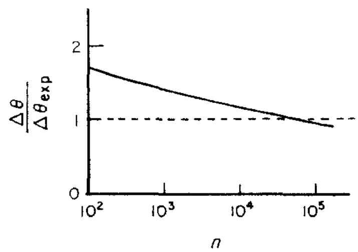

FIG. 4. The ratio between the calculated and the measured $\Delta\theta$.

3. CONCLUSIONS

The phenomena of the nucleate boiling heat transfer consist of two parts with quite different characters. One is the origination of bubble nuclei at fine cavities on heating surfaces, which has a non-reproducible character from the macroscopic point of view and cannot be treated with the ordinary method of the physics of continuum. The other is the heat transmission by convective liquid flow in the thermal boundary layer induced by the growing and rising of bubbles; this may be explained by the ordinary method of the continuum-physics. It would be reasonable that these two parts are treated separately. By establishing a hydrodynamic model of the nucleate boiling, the latter part of the problem is analysed by the author and a theoretical formula obtained. The theoretical result being in good agreement with experimental data, the model proposed by the author may be considered to have reality.

If we want to understand the whole process including the former part with non-reproducible character, we must enter into the mechanism of the creation of bubble-nuclei which relates closely to the fine cavities distributed irregularly on heating surfaces and may be treated only by using statistical methods. Thus results obtained in this paper may be said to show the limit which can be approached by the physics of continuum.

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APPENDIX. LIQUID MOTION INDUCED BY BUBBLES GROWING AND RISING

A.1 *The general aspect of the liquid motion induced by bubbles*

Isshiki and Tamaki [11] took a Schlieren motion picture of bubbles rising from a heated wire and noticed that every bubble trails its own tail, as it were, of a mushroom type. The author [12] proposed that the tails may be explained by a Lagrangian motion of thermal boundary layer which exists at first near the heated wire, and calculated the motion theoretically. The changing shape of the tail of the bubble with time are illustrated in Fig. 5, which are in good agreement with those obtained by the Schlieren motion picture.

In the nucleate boiling from a heated wire, as we have seen, the liquid flow induced by the bubble-motion plays a very important role. In the nucleate boiling from a flat heating surface, as in our case, the liquid-flow induced by the bubble motion is expected to be also very important. The condition in the case of the latter,

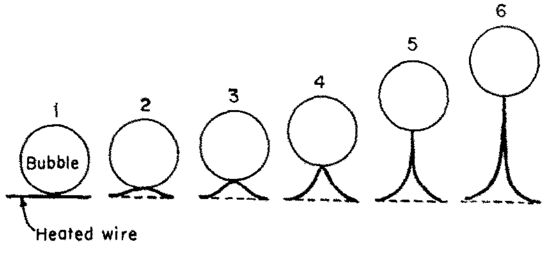

FIG. 5. The changing shape of the tail of a bubble with time.

however, is somewhat different from the case of the former. In the case of a heated wire the flow is induced in the whole space as shown in Fig. 6a, but in the case of a heated flat plate the flow is

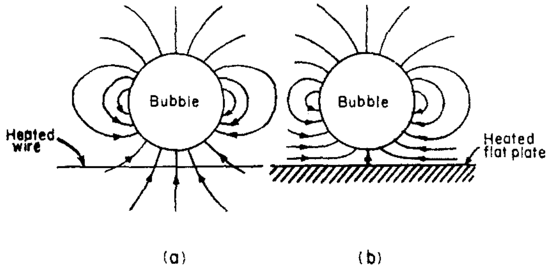

FIG. 6. Stream lines around a bubble.
(a) A bubble is leaving a heated wire.
(b) A bubble is leaving a heated flat plate.

restricted to the upper half of the space as shown in Fig. 6b, and the liquid near the flat plate is expected to flow parallel to the plate. Consequently the liquid in the thermal boundary layer over the plate is expected also to move toward nucleation site. Going into detail, we see that at first the liquid in the thermal boundary layer moves away from the nucleation site with the bubble-growing and then moves back toward it with the bubble-rising. The resultant displacement, that is, the algebraic sum of these two displacements with opposite directions, has the direction toward the nucleation site. Accompanied by the to-and-fro motion of the liquid, an unsteady velocity boundary layer is expected to be generated on the heating surface. Its thickness can be easily estimated and is known to be much smaller than that of the thermal boundary layer. Thus the velocity boundary layer is ignored in the following theoretical analysis and the flow of the liquid is treated as a potential flow.

A.2 *Liquid motion induced by bubble-growing*

The liquid surrounding a bubble which is growing on a heating surface is pushed away. Taking a reflected image into account as shown in Fig. 7, the velocity potential ϕ_1 of the liquid motion induced by the bubble-growing on the heating surface can be expressed approximately as follows:

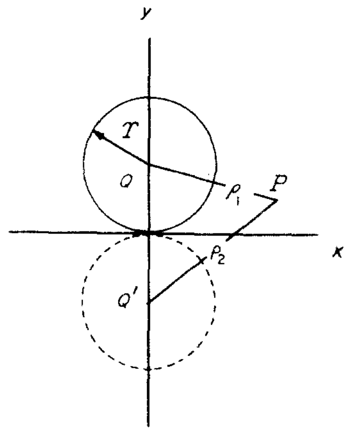

FIG. 7.

$$\phi_1 = b\left[\frac{1}{\rho_1} + \frac{1}{\rho_2} + 0{\cdot}41\,\frac{r(y-r)}{\rho_1^3} - 0{\cdot}41\,\frac{r(y+r)}{\rho_2^3}\right] \quad (11)$$

where r is the radius of the growing bubble, which is a function of time t, ρ_1 is the distance from an arbitrary point P to a source Q and ρ_2 is that from P to Q' which is the reflected image of Q. The third and fourth terms of the right-hand side of (11) are those which correct the deformation of the bubbles deviating from spheres caused by the existence of the reflecting image; b depends on the time rate of the growing of the bubble and can be determined approximately as follows. The velocity potential for an isolated bubble, without reflected image, is expressed as $\phi_{is} = b/\rho$. The liquid velocity on the surface of the bubble is

$$u_r = \left(\frac{\partial \phi_{is}}{\partial \rho}\right)_{\rho = r} = -\frac{b}{r^2},$$

which is equal to $(\mathrm{d}r/\mathrm{d}t)$. Namely

$$b = -r^2\frac{\mathrm{d}r}{\mathrm{d}t}. \quad (12)$$

From (11) and (12) the velocity field in the liquid is determined. Accordingly, the motions of a liquid particle with time can be determined from these equations and the locus of the particle can be obtained. The co-ordinates x, y of a liquid particle can be calculated through the integration of $u_1 = \partial\phi_1/\partial x$ and $v_2 = \partial\phi_1/\partial y$ with time t as follows:

$$\left.\begin{aligned} x &= x_o + \int_o^t u_1(x, y)\,\mathrm{d}t \\ y &= y_o = \int_o^t v_1(x, y)\,\mathrm{d}t \end{aligned}\right\} \qquad (13)$$

where x_o and y_o are their values at the instant $t = 0$ when the nucleus of the bubble is created.

A.3 *Liquid motion induced by bubble-rising*

The velocity potential ϕ_2 (observed from a point at rest) and the stream function ψ_2 (observed from a point fixed to the moving bubble) in the liquid in which a bubble with a radius r_o is rising from a heating surface with a velocity U_o = constant can be expressed as follows:

$$\phi_2 = -\frac{U_o}{2}(y - h)\left(\frac{r_o}{\rho_1}\right)^3 + \frac{U_o}{2}(y + h)\left(\frac{r_o}{\rho_2}\right)^3 \qquad (14)$$

$$\psi_2 = -\pi\, U_o\, x^2 \left[1 - \left(\frac{r_o}{\rho_1}\right)^3 + \left(\frac{r_o}{\rho_2}\right)^3\right] \qquad (15)$$

where ρ_1 is the distance from an arbitrary point P to a doublet Q, ρ_2 is that from P to Q' which is the reflected image of Q, and h is the distance from the heating surface to Q as shown in Fig. 8; h is, of course, a function of time. The form of the bubble is determined from the following equation

$$1 - \left(\frac{r_o}{\rho_1}\right)^3 + \left(\frac{r_o}{\rho_2}\right)^3 = 0. \qquad (16)$$

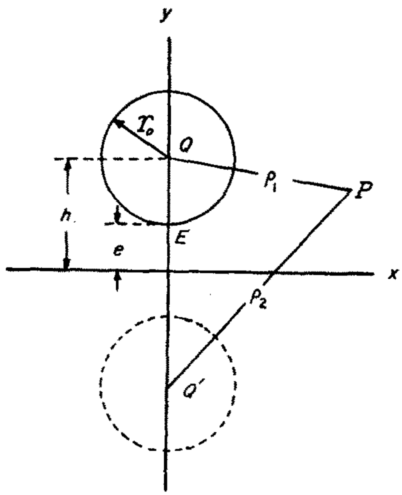

FIG. 8.

When the doublet Q is sufficiently apart from the heating surface, the third term of (16) is small and the form determined by (16) is approximately spherical. When Q is near the surface, the form deviates from a sphere. Practically, the deviation is not very large, as explained in the footnote.† As the surface expressed by (16) deviates from a sphere with the decreasing of h, even if the deviation is small, the rising velocities of every portion on the surface (16) are different from each other. We define U_o as the rising velocity of the lowest point E of the surface (16) in Fig. 8. Taking e as the distance from E to the heating surface, we have

$$\frac{\mathrm{d}e}{\mathrm{d}t} = U_o. \qquad (17)$$

From (16)

$$1 - \left(\frac{r_o}{h - e}\right)^3 + \left(\frac{r_o}{h + e}\right)^3 = 0. \qquad (18)$$

Thus, the velocity field can be determined as a function of time from equations (14), (17) and (18). The co-ordinates of the liquid particle x and y can be calculated through integration $u_2 = \partial\phi_2/\partial x$ and $v_2 = \partial\phi_2/\partial y$ with time as follows:

$$\left.\begin{aligned} x &= x_1 + \int_{t_1}^t u_2(x, y)\,\mathrm{d}t \\ y &= y_1 + \int_{t_1}^t v_2(x, y)\,\mathrm{d}t \end{aligned}\right\} \qquad (19)$$

where x_1 and y_1 are their values at the instant of t_1 when the bubble leaves the heating surface.

A.4 *The motion of liquid particles which exist at first on the outer edge of the thermal boundary layer*

In this section the motions of liquid particles which exist at first on the outer edge of thermal boundary layer are traced theoretically. If we consider the fluid motion only which occurs in the vicinity of the heating surface, the velocities can be expressed rather simply. In the process of

† Strictly speaking, the surface expressed by (16) cannot contact with the heating surface without tending of h to 0. As h tends to 0, the deviation of the surface from a sphere becomes remarkable. But, if h is taken as small as 0·7 r_o, the distance from the lower surface of (16) to the heating surface becomes smaller than 0·02 r_o. The surface in this case can be considered to contact with the heating surface approximately and its deviation from a sphere is not large.

the bubble-growing, u_1 and v_1 can be obtained from equation (11) under the condition $y \ll r$ as follows:

$$\left.\begin{aligned} u_1 &= b\,x\left[-\frac{2}{(r^2+x^2)^{\frac{3}{2}}}+\frac{2{\cdot}46\,r^2}{(r^2+x^2)^{\frac{5}{2}}}\right] \\ v_1 &= b\,y\left[-\frac{2}{(r^2+x^2)^{\frac{3}{2}}}\right. \\ &\qquad \left.+\frac{13{\cdot}4\,r^2}{(r^2+x^2)^{\frac{5}{2}}}-\frac{12{\cdot}3\,r^4}{(r^2+x^2)^{\frac{7}{2}}}\right]. \end{aligned}\right\} \quad (20)$$

In the process of the bubble-rising, u_2 and v_2 can be obtained from (14) under the condition $y \ll h$ as follows:

$$\left.\begin{aligned} u_2 &= -3U_o\,r_o^3\,h\,x\,\frac{1}{(h^2+x^2)^{\frac{5}{2}}} \\ v_2 &= U_o\,r_o\,h\,y\left[-\frac{9}{(h^2+x^2)^{\frac{5}{2}}}+\frac{15\,h^2}{(h^2+x^2)^{\frac{7}{2}}}\right]. \end{aligned}\right\} \quad (21)$$

x components of the velocities u_1 and u_2 are independent of y. y components of the velocities v_1 and v_2 are products of y and functions of x only.

Denoting the thickness of thermal boundary layer as δ, the loci of the motion of the liquid particles which were at first on the outer edge of the thermal boundary layer are obtained from (13), (19), (20) and (21). The loci thus obtained have such a shape as shown in Fig. 2. The locus shown in the figure is correct in shape, but is exaggerated in size. Its actual proportion to the thickness of thermal boundary layer δ is far smaller than that shown in the figure. A liquid particle which exists at first on the outer edge of the thermal boundary layer P was washed away to the point Q with the growing of a bubble. Then the liquid particle comes back to the point R with the rising of the bubble from the heating surface to a point at infinity. As a matter of fact, if the height of the bubble from the heating surface h exceeds 5 r_o, even though the bubble does not go to the point at infinity, the liquid particle can be considered practically to reach the point R. And the horizontal location of the point R is nearer to the nucleation site than the original point P. We denote the vertical displacement from P to R by δ_*. The computed values of δ_*/δ vs. x/r_o are shown in Fig. 9. The relation can be approximated as follows:

$$\frac{\delta_*}{\delta} = 0{\cdot}126\left(\frac{x}{r_o}\right)^{-2{\cdot}32}. \quad (22)$$

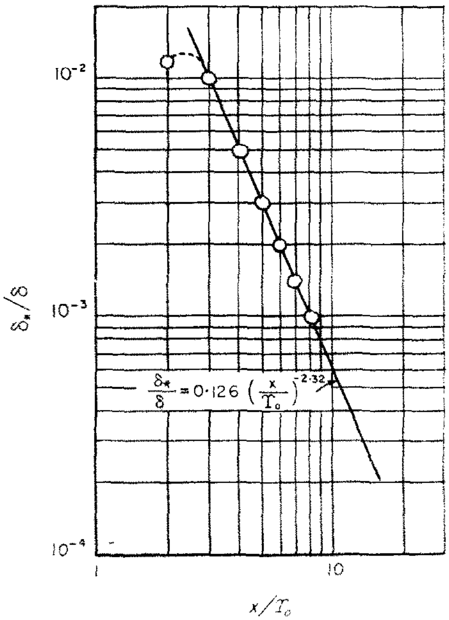

FIG. 9. The variation of δ_*/δ with x/r_o. Marks ○ show the calculated values.

A.5 *The effect of many nucleation sites*

In the last section the case of an isolated nucleation site is considered. In this section we deal with the cases where many nucleation sites distribute on the heating surface; δ_*/δ for these cases can be obtained by summing up all the contributions from each nucleation site on the heating surfaces.

We assume that the nucleation sites are arranged on the heating surface in the equilateral triangular pattern as shown in Fig. 10. The length l shown in the figure is taken as a characteristic length to determine the distance between two neighbouring nucleation sites. In other words l determines the surface density of nucleation sites of the heating surface. We also assume that the sizes of rising bubbles are all the same independently of nucleation sites and that the periods of the bubble formation in all the nucleation sites are also identical.† Consequently the total vertical displacement of a liquid particle during one period T can be obtained simply through the summation of the

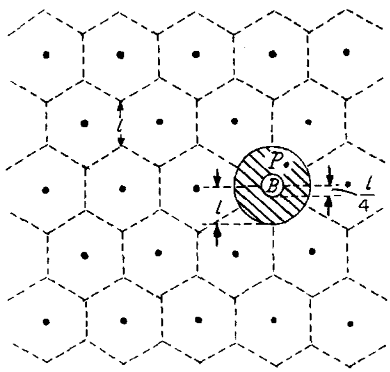

FIG. 10. The pattern of distribution of nucleation sites.

contributions from all the nucleation sites. Thus the total vertical displacement at an arbitrary point P on the heating surface can be written using the equation (22) as follows:

$$\left(\frac{\delta_*}{\delta}\right)_P = \sum_\nu 0{\cdot}126 \left(\frac{x_\nu}{r_o}\right)^{-2{\cdot}32} \tag{23}$$

where x_ν is the distance from P to an arbitrary nucleation site ν, and the summation $\sum$ is carried out over all the nucleation sites ν on the heating surface.

If l is elongated or contracted, x_ν varies with it proportionally. Thus we can put as follows:

$$x_\nu = c_\nu\, l \tag{24}$$

where c_ν is a proportional constant.

The number of nucleation sites per unit area n varies with the length l. The relation between n and l is expressed as follows;

$$n = \frac{2}{3\sqrt{(3)}}\,\frac{1}{l^2}. \tag{25}$$

Substituting (24) and (25) into (23), we get

$$\left(\frac{\delta_*}{\delta}\right)_P = 0{\cdot}383\,(n\,r_o^2)^{1{\cdot}16} \sum_\nu c_\nu^{-2{\cdot}32}. \tag{26}$$

† The phases of the bubble-formation in each nucleation site need not always be equal.

$(\delta_*/\delta)_P$ thus obtained is, of course, a function of the position P.

Actually the summation of the series $\sum_\nu c_\nu^{-2{\cdot}32}$ was performed in the following way. The thirty terms caused by the nearest thirty nucleation sites from P are summed up directly. These thirty, nucleation sited, are in the inside of a circle with a radius of about 5 l and P as its centre. Concerning the terms caused by the nucleation sites in the outside of the circle, the terms are calculated considering approximately that the nucleation sites are distributed continuously over the heating surface, and converting the summation into an integral. Then, the summation of the first thirty terms and the integral are summed up together.

In the next place we want to take the mean value of $(\delta_*/\delta)_P$ over the heating surface. This can be obtained through taking a mean value over a hexagonal portion of the heating surface around a nucleation site B in Fig. 10. The mean value is indicated by $(\delta_*/\delta)_{\text{mean}}$. In the practical calculation, an annular area as shown in Fig. 10 is used instead of the hexagonal area for the sake of simplicity. The outer radius of the annular area is equal to l and the inner one is $l/4$. The reason why the inner circular area with radius $l/4$ is hollowed out is that the relation shown in (22) does not hold in the vicinity of the nucleation site.‡ The error which is expected to occur through the hollowing out of the inner circular area is, however, supposed to be so small that it can be neglected, because its area is only one-sixteenth of that of the outer circle.

The mean value thus obtained is expressed as follows:

$$\left(\frac{\delta_*}{\delta}\right)_{\text{mean}} = 5{\cdot}5\,(n\,r_o^2)^{1{\cdot}16}. \tag{27}$$

For example, in the case where $r_o = 2$ mm and $n = \frac{1}{4}$ cm^{-2}, $(\delta_*/\delta)_{\text{mean}} = 0{\cdot}026$.

A.6 *Qualitative explanation of the relation found by Jakob*

The liquid motion in the surrounding of a rising bubble is caused by the following two

‡ In truth the region where (22) does not hold depends upon r_o and is independent of l. In the text we assumed that the portion where (22) does not hold is so small that it is in the inside of the inner circle with a radius $l/4$.

actions. One is the pushing-up of the liquid in the topside of the bubble and the other is the filling-up of the cavity left by the moving bubble with the liquid. If many bubbles would rise from a nucleation site one after another at a very small spacing with each other, the greater part of the cavity left by a preceding bubble would be filled up with the liquid portion pushed up by the succeeding bubble and consequently the liquid at some distances from the line of the bubbles would scarcely move. In this case the liquid motion in the thermal boundary layer would not occur, so that the formation of bubbles could not take place. Thus it can be said, at least, that a necessary condition for the occurrence of the formation of bubbles is the sufficient largeness of the spacing L between two neighbouring bubbles. To be exact, the ratio of the spacing L to the radius of bubbles r_0 cannot be smaller than a certain lower limit c.

$$\frac{L}{r_0} \geqslant c = \text{const.} \tag{28}$$

The frequency of bubble-formation at a nucleation site can be written by

$$f = \frac{U_0}{L} \tag{29}$$

where U_0 is the velocity of bubble-rising.

From (28) and (29)

$$f r_0 \leqslant \frac{U_0}{c}. \tag{30}$$

From the experiment performed by Datta [13], U_0 can be assumed approximately to be constant, so that

$$f r_0 \leqslant \text{const.} = c_2. \tag{31}$$

In the next place we should like to explain that only the sign of equality in (31) is possible. It is obvious from (4) that n, f and r_0 must increase with the heat flux q. Among these three variables, only n has a different character from two other variables f and r_0. n cannot increase continuously but takes only discrete values, that is, n increases stepwise. We denote the area of the heating surface by S. The number of the nucleation sites on the heating surface is expressed by $N = nS$. Now we suppose a case in which the heat-flux q would increase gradually. f and r_0 would increase with it also gradually. Only N could not increase until the increment of the heat-flux amounts to a certain finite value Δq. At the instant when the increment of the heat-flux comes up to this definite value Δq, N increases by 1, that is, jumps to $N + 1$. Consequently $n = N/S$ jumps to $(N + 1)/S$. The amount of heat-flux increment Δq required to make N jump to $N + 1$ is known to be considerably large from the Nishikawa's experiment [2]. Returning to the subject, we consider the case again where the heat-flux q is increasing gradually; $f r_0$ increase with q and get nearer to the critical value c_2 in (31). As Δq required for jumping of N by 1 is considerably large, it is considered to be probable that $f r_0$ becomes equal to c_2 just before N jumps to $N + 1$. In this way the equality

$$f r_0 = c_2 \tag{32}$$

is always satisfied. This is the very relation that was found by Jakob [1].

Résumé—Dans l'ébullition nucléée, la bulle qui prend naissance sur la surface chauffée grossit, quitte la paroi et s'élève. Le mouvement induit dans la couche limite thermique au cours de ce processus est calculé et on obtient le flux de chaleur transporté par ce mouvement liquide. Le flux de chaleur ainsi calculé est égal à celui transmis, par conduction, de la surface chauffée au liquide et à la chaleur latente enlevée par la bulle par unité de temps. A partir de ces relations, on obtient la formule théorique suivante

$$\Delta\theta = 0{,}114\, n^{-\frac{1}{4}}\, q^{\frac{2}{3}}$$

où $\Delta\theta$ est la différence entre la température de la surface chauffante et la température de saturation, n est le nombre de "sites de formation de bulles" par unité de surface et q le flux de chaleur moyen. Elle a une forme voisine de celle de la formule expérimentale obtenue à partir des mesures de Nishikawa

$$\Delta\theta_{\text{exp}} = 0{,}448\, n^{-\frac{1}{6}}\, q^{\frac{2}{3}}$$

Les valeurs numériques calculées à partir de ces deux formules sont en bon accord.

Zusammenfassung—Beim Blasenverdampfen bildet sich an der Heizfläche an einem Keim eine Blase, die wächst, sich von der Fläche ablöst und aufsteigt. Die während dieses Prozesses in der thermischen Grenzschicht ausgelöste Flüssigkeitsbewegung wird berechnet und man erhält den durch die Bewegung der Flüssigkeit zur Keimstelle hin hervorgerufenen Wärmestrom. Der so ermittelte Wärmefluss ist gleich dem von der Heizfläche an die Flüssigkeit durch Leitung übertragenen und gleich der von den Blasen pro Zeiteinheit abgeführten latenten Wärme. Aus diesen Beziehungen erhält man folgende theoretische Formel

$$\Delta\theta = 0{,}114\, n^{-\frac{1}{4}} q^{\frac{2}{3}}$$

mit $\Delta\theta$ als Temperaturdifferenz zwischen Heizfläche und Sattdampf, n der Anzahl der Keime pro Flächeneinheit und q dem mittleren Wärmefluss. Diese Formel ist ihrer Form nach der von Nishikawa gefundenen Gebrauchsformel

$$\Delta\theta_{\mathrm{exp}} = 0{,}448\, n^{-\frac{1}{6}} q^{\frac{2}{3}}$$

ähnlich. Die nach beiden Formeln errechneten Zahlenwerte stehen miteinander in leidlich guter Übereinstimmung.

Аннотация—При пузырьковом кипении пузырек, образующийся в определенном месте на поверхности нагрева, растет, отрывается от нее и поднимается вверх. В статье приводится расчет движения жидкости, возникающего в тепловом пограничном слое во время этого процесса, а также теплового потока к месту образования пузырька. Вычисленная таким образом интенсивность теплопереноса включает в себя тепловой поток теплопроводности от поверхности нагрева к жидкости и скрытую теплоту, уносимую пузырьками в единицу времени.

Из этих соотношений получена следующая расчетная формула:

$$\Delta\theta = 0{,}114\, n^{-\frac{1}{4}} q^{\frac{2}{3}}$$

где $\Delta\theta$—разность температуры поверхности нагрева и температуры насыщения, n-число мест образования пузырьков на единипу площади и q—средняя величина теплового потока. Уравнение по своей форме близко к экспериментальной зависимости

$$\Delta\theta_{\mathrm{exp}} = 0{,}448\, n^{-\frac{1}{6}} q^{\frac{2}{3}}$$

Численные величины, полученные из этих двух формул, хорошо согласуются между собой.